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Creation of periodic and chaotic attractors using blending technique

Vinod Patidar and K K Sud*

Department of Physics, College of Science Campus, M L S University, Udaipur-313 002, Rajasthan, India

E-mail : kksud@yahoo.com

Abstract : In this communication, we demonstrate that by using blending of chaotic attractor with a limit cycle attractor generated by the similar system operating in the periodic regime of oscillations (may be weak, period-1, period-2 *etc.*), one can create various periodic and chaotic attractors having properties intermediate between the chaotic and periodic attractors, which were initially used for the blending mechanism. Basically it is the mutual coupling mechanism between the phase variables of chaotic and periodic oscillators of same kind, which does not rely on the knowledge of the system model equations and the system parameters are not changed explicitly. The technique is expected to be useful for those physical systems in which direct accessibility of its parameters is either difficult or not possible. We present the results of our numerical simulations on Duffing oscillator and Jerk dynamical systems, which represent the plasma oscillations and WINDMI model of the earth's magnetosphere respectively.

Keywords : Blending technique, chaos suppression, designer chaotic signals, mutual coupling.

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Dynamical chaos is very interesting phenomenon and it has been detected in a large number of nonlinear dynamical systems of different physical nature. In many situations chaos is undesirable phenomenon, which may lead to irregular operation in physical systems. Thus from a practical point of view one would like to convert chaotic solutions into periodic limit cycle or fixed point solutions. On the other hand, there has been growing interest to use chaos profitably by synchronizing chaotic orbits [1,2] due to its potential application in secure communication [2–7]. It has also become important in order to evaluate the characteristics of physical systems by using scalar observations even when the output is chaotic.

In general, we wish to control chaotic oscillations in some systems of practical interest whereas for the other requirements, we desire to produce chaotic signals having specific properties. Till now, we often focus on the limited number of available chaotic systems instead of producing a chaotic signal having specific properties for the chosen purpose. It would be quite interesting to produce various chaotic signals having desired properties from the available chaotic source. If we do not have direct access to the system parameters of the available physical system then the problem of designing chaotic signals of desired properties from it, presents a big challenge. However a number of algorithms [8–10] have been suggested in the literature for controlling chaos but the designing of chaotic signal having specific properties for the chosen purpose is still a challenge. At this time no general mathematical solution of this problem exists in the literature. We propose a technique, which is expected to provide a practical solution to this problem. Recently Patidar *et al* [11] have proposed an algorithm for suppression of chaos in which authors were able to suppress chaotic oscillations by means of establishing a mutual coupling of the chaotic system with a similar system operating in a weakly periodic regime of oscillations. The technique developed by Patidar *et al* does not require the knowledge of the system model equation and the system parameters are not changed during the control task. In this communication, we present the generalization of the algorithm developed by Patidar *et al* [11] through which we are able to suppress the chaos as well as design the chaotic signal of specific

*Corresponding Author

properties from the available chaotic signal. We particularly report that by using blending of chaotic oscillator with a similar oscillator operating in a periodic regime of oscillations (may be weak periodic, period-1, period-2 limit cycles *etc.*), one can create various periodic and chaotic signals having properties intermediate between the chaotic and periodic signals, which were initially used for the blending mechanism. Basically it is a mutual coupling mechanism between the phase variables or combination of phase variables (which are accessible in actual physical systems) of the chaotic and periodic oscillators of same kind. We designate this mutual coupling transformation as '*symmetric blending*' of chaotic and periodic oscillators of similar kind because of the following reasons :

(i) The mutual coupling is done for any state variable or combination of state variables, which are accessible in the actual physical systems *i.e.* it does not require the knowledge of the system model equation.

(ii) The mutual coupling is symmetric *i.e.* the chosen state variable or combination of state variables for the mutual coupling in periodic and chaotic oscillators are the counter parts.

(iii) The mutual coupling does not change any of the system parameters explicitly *i.e.* no need to have direct access of any of the system parameters.

In general any physical oscillator can be represented as follows :

$$O(\ddot{x},\dot{x},x,S(x),\mu)=0\,, \tag{1}$$

here x, $\dot{x}$ and $\ddot{x}$ are the state variable and its derivatives respectively, $S(x)$ is state variable or combination of state variables (*i.e.* x or x^2 or $\sin x$ *etc.*) which is accessible in the actual physical system and $\mu \equiv (\mu_1,\mu_2,......\mu_n)$ is the set of system parameters. We are considering the situation in which, we do not have access to any of the system parameters $(\mu_1,\mu_2,......\mu_n)$ and eq. (1) possesses chaotic and periodic behaviours at least for certain range of system parameters.

For the proposed blending mechanism, we start with two oscillators of same kind, one of them is in chaotic regime of oscillation and another one is in periodic regime of oscillation as stated below :

$$O(\ddot{x},\dot{x},x,S(x),\mu_p)=0\,, \tag{2}$$

$$O(\ddot{y},\dot{y},y,S(y),\mu_c)=0\,, \tag{3}$$

here μ_p and μ_c represent the set of system parameters corresponding to periodic and chaotic oscillations respectively. We have assigned x-variable and y-variable for the chaotic and periodic oscillators respectively.

After establishing the proposed blending (*i.e.* mutual coupling) the eqs. (2) and (3) will take the forms :

$$O(\ddot{x},\dot{x},x,\{c*S(y)+(1-c)*S(x)\},\mu_p)=0\,, \tag{4}$$

$$O(\ddot{y},\dot{y},y,\{c*S(x)+(1-c)*S(y)\},\mu_c)=0\,, \tag{5}$$

here c represents the mutual coupling strength *i.e.* percentage of blending between the chaotic and periodic oscillators and by varying the percentage of blending one can make a choice for periodic or chaotic signal as required for the specific purpose. In Figure 1, we have shown the schematic representation of the proposed symmetric blending mechanism between chaotic and periodic oscillators. In the next paragraph, we present

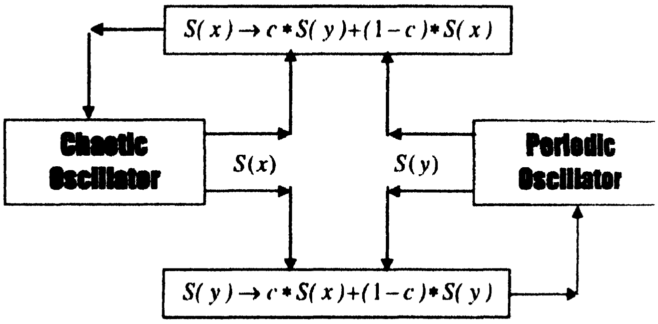

Figure 1. Schematic representation of blending of chaotic and periodic oscillators of same kind. $S(x)$ and $S(y)$ are the state variables or combination of state variables in chaotic and periodic oscillators respectively, which are accessible in the actual physical systems and c is the percentage of blending (*i.e.* mutual coupling strength) between the oscillators.

results of our numerical simulations using the proposed blending mechanism on Duffing oscillator and jerk dynamical systems.

Duffing oscillator is a second order nonlinear differential equation, which can be represented as :

$$\ddot{x}+d\dot{x}-x(1-\underline{x}^2)=f\cos\omega t\,, \tag{6}$$

This equation represents forced vibrations of a buckled beam [12], where x is the lateral motion of the beam. Eq. (6) has also been used to study the motion of a particle in two well potential [13] as well as plasma oscillations [14] and it exhibits period doubling route to chaos in the parameter space (f,d). For numerical simulations, we have assumed that the underlined term (*i.e.* $S(x)=x^2$) is accessible in the actual physical system. The establishment of the proposed blending in Duffing oscillator leads to the following equations :

$$\ddot{x}+d_1\dot{x}-x[1-\underline{\{cy^2+(1-c)x^2\}}]=f_1\cos\omega t, \tag{7a}$$

$$\ddot{y}+d_2\dot{y}-y[1-\underline{\{cx^2+(1-c)y^2\}}]=f_2\cos\omega t, \tag{7b}$$

In Figure 2, we show the results of our numerical simulations of blending of chaotic and weak periodic Duffing oscillators (f_1 = 0.37, d_1 = 0.5, f_2 = 0.05, d_2 = 0.05 and ω = 1.0). Each row of frames shows the

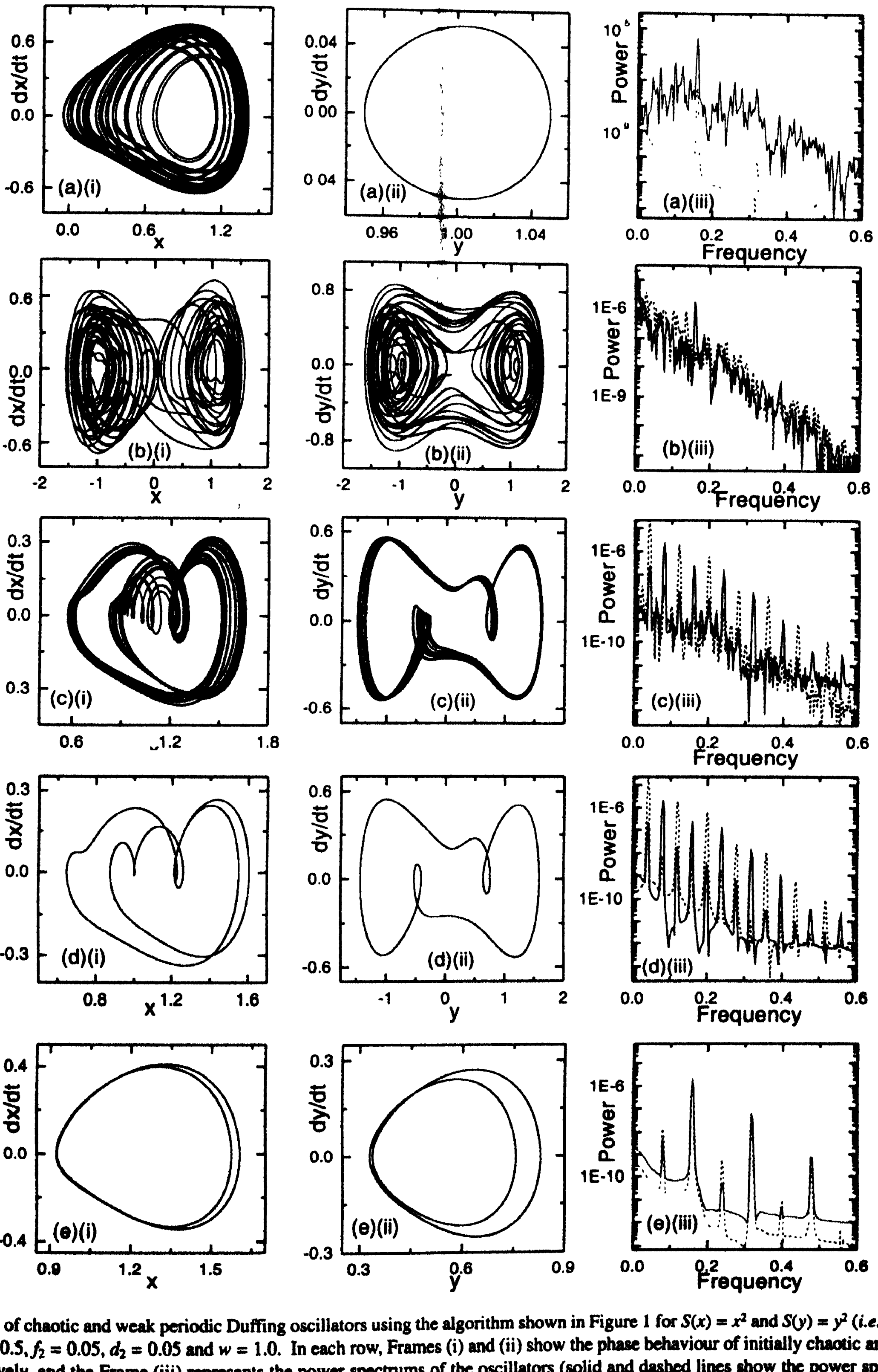

Figure 2. Blending of chaotic and weak periodic Duffing oscillators using the algorithm shown in Figure 1 for $S(x) = x^2$ and $S(y) = y^2$ (*i.e.* eqs. (7a) and (7b)) and $f_1 = 0.37$, $d_1 = 0.5$, $f_2 = 0.05$, $d_2 = 0.05$ and $w = 1.0$. In each row, Frames (i) and (ii) show the phase behaviour of initially chaotic and periodic Duffing oscillators respectively, and the Frame (iii) represents the power spectrums of the oscillators (solid and dashed lines show the power spectrums of initially chaotic and periodic Duffing oscillators respectively). (a) $c = 0$, (b) $c = 0.3$, (c) $c = 0.473$, (d) $c = 0.477$ and (e) $c = 0.484$.

behaviour of the coupled system (eqs. (7a) and (7b)) for a specific value of the mutual coupling strength (c). Particularly, first row (indexed by (a)), second row (indexed by (b)), third row (indexed by (c)), fourth row (indexed by (d)) and fifth row (indexed by (e)) represent the behaviour of the coupled system (eqs. (7a) and (7b)) for c = 0, c = 0.3, c = 0.473, c = 0.477 and c = 0.484 respectively. In each row, Frames (i) and (ii) show the phase behaviour of initially chaotic and periodic oscillators respectively. The Frame (iii) represents the power spectrums of both the oscillators (solid and dashed lines are respectively for initially chaotic and periodic oscillators) and also shows the dominant frequency components present in the signals shown in Frames (i) and (ii). We observe from Figure 2 that as we increase the percentage of blending (mutual coupling strength (c)) between the chaotic and weak periodic Duffing oscillators, we obtain the various chaotic and periodic attractors which inherently exist in the oscillator system (for different sets of parameters) but due to unavailability of the direct access to its system parameters, can not be produced.

We would also like to mention here that we are imposing the blending (mutual coupling) externally for the practical purpose so obviously we have the access to the coupling strength and hence one can precisely fix the percentage of blending and thus in a position to produce the chaotic or periodic signal of the desired property.

We have also tested the proposed blending technique on another recently investigated simple chaotic system involving jerk equation [15–18]. Jerk equation is an ordinary third order nonlinear differential equation in one real scalar dynamical variable. The functional form of the jerk equation is $\dddot{x} = J(x, \dot{x}, \ddot{x})$, here $\dddot{x}$ is rate of change of acceleration and is called jerk. We have considered the following form of the jerk equation :

$$\dddot{x} + A\ddot{x} + \dot{x} - B(\underline{x}^2 - 1) = 0 \,. \tag{8}$$

This equation also shows period doubling route to chaos in the parameter space (A,B) [15,19]. We would also like to mention here that eq. (8) also represents the simplified model (with reasonable approximations) of the six dimensional solar wind driven magnetosphere ionosphere (WINDMI) model [20–22]. The difference is only in the form of nonlinearity *i.e.* $B(x^2 - 1)$ is to be replaced by $B(\tan h\, x + C)$. Similar to the earlier example of Duffing oscillator, we have assumed that the underlined term (*i.e.* $S(x) = x^2$) is accessible in the actual physical system and the proposed blending leads to the following coupled equations :

$$\dddot{x} + A_1\ddot{x} + \dot{x} - B_1[\{cy^2 + (1-c)x^2\} - 1] = 0, \tag{9a}$$

$$\dddot{y} + A_2\ddot{y} + \dot{y} - B_2[\{cx^2 + (1-c)y^2\} - 1] = 0, \tag{9b}$$

In Figure 3, we show the results of our numerical simulations of blending of chaotic and periodic (period-2) jerk dynamical systems (A_1 = 0.6, B_1 = 0.58, A_2 = 0.7 and B_2 = 0.58). In this case first row (indexed by (a)), second row (indexed by (b)), third row (indexed by (c)), fourth row (indexed by (d)) and fifth row (indexed by (e)) represent the behaviour of the coupled system (eqs (9a) and (9b)) for c = 0, c = 0.1, c = 0.2, c = 0.3 and c = 0.37 respectively. It is also clear from Figure 3 that the proposed blending technique is able to produce various chaotic and periodic signals by simply varying the percentage of blending between the chaotic and periodic (period-2) jerk dynamical systems.

In both cases (Duffing oscillator and jerk dynamical systems) investigated by us, we have taken x^2 as the accessible term and considered a few selected set of parameters. However, we have found in our numerical simulations that the proposed technique is also successful if we choose other terms (x, x^2 *etc.*) as accessible term (in a real system the selection of the term will depend on the physical accessibility) as well as consider other set of parameters. For brevity, we have not given the figures corresponding to other terms and set of parameters here.

In summary, by generalizing the chaos suppression algorithm [11], we have attempted to tackle two problems (*i.e.* chaos suppression and design of chaotic signals from the available chaotic signal) with a single algorithm and successfully tested it on two oscillators. However, a number of algorithms exist in the literature for the suppression of chaos but the proposed blending technique is easily implementable without having the knowledge of system model equation as well as without disturbing the system parameters explicitly. In addition, the proposed blending technique is also able to produce various designer chaotic signals, so it is expected to be more advantageous for various cryptographic and secure communication [2–7] purposes, where the chaotic signals of desired properties are required.

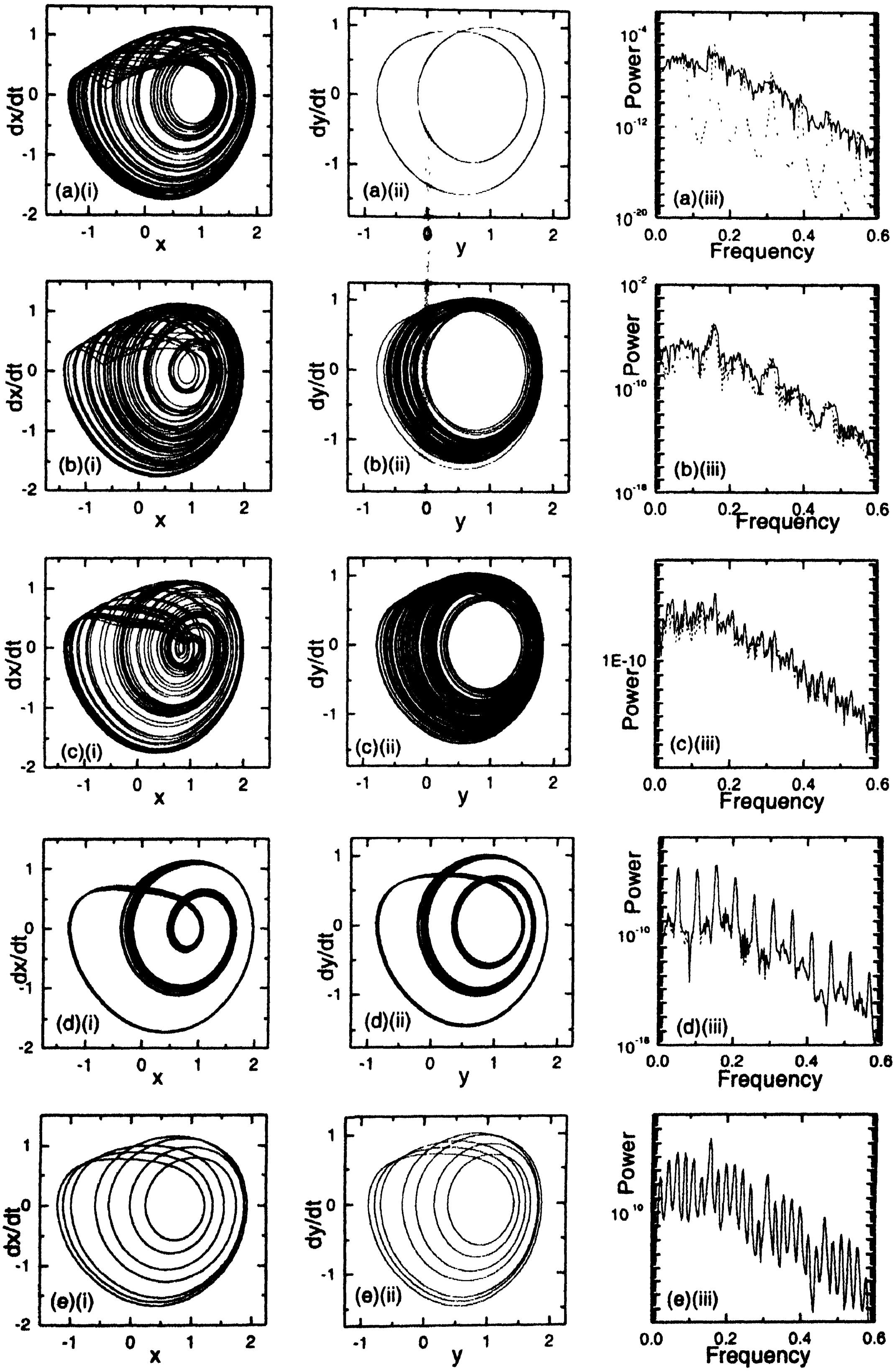

Figure 3. Blending of chaotic and weak periodic (period-2) jerk dynamical systems using the algorithm shown in Figure 1 for $S(x) = x^2$ and $S(y) = y^2$ (*i.e.* eqs. (9a) and (9b)) and $A_1 = 0.6$, $B_1 = 0.58$, $A_2 = 0.7$ and $B_2 = 0.58$. In each row, Frames (i) and (ii) show the phase behaviour of initially chaotic and periodic jerk dynamical systems respectively, and the Frame (iii) represents the power spectrums of the systems (solid and dashed lines show the power spectrums of initially chaotic and periodic jerk dynamical systems respectively). (a) $c = 0$, (b) $c = 0.1$, (c) $c = 0.2$, (d) $c = 0.3$ and (e) $c = 0.37$.

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